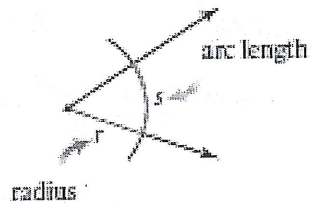
13.2 – Radian Measure

One way to measure an angle is in degrees. There are __360__ degrees in a circle.

Another way to measure an angle is in **radians**. There are __2π__ radians in a circle.

Like degrees, radians are used to measure angles and rotations. The radian measure of an angle can be found by constructing *any* arc intercepted by that angle, with its center at the vertex of the angle, and dividing the length of that arc by its radius.

$$\textit{radian measure} = \frac{\textit{arc length}}{\textit{radius}}$$



The circumference of a circle depends on the length of its radius. Using the radian measure definition and the formula $C = 2\pi r$, you will find that the radian measure of a full circle, or 360° rotation, is $\frac{\text{arc length}}{\text{radius}} = \frac{2\pi r}{r} = 2\pi$ radians.

Converting from degrees to radians:

$$\frac{\textit{angle in degrees}}{360} = \frac{\textit{angle in radians}}{2\pi} \qquad \frac{\textit{angle in degrees}}{180} = \frac{\textit{angle in radians}}{\pi}$$

Example 1: What radian measure is equivalent to 50°?

$\frac{50}{360} = \frac{\theta}{2\pi}$

$\frac{360\theta}{360} = \frac{100\pi}{360}$

$\theta = \frac{5\pi}{18}$ radians $\approx .873$ radians

220°?

$\frac{220}{360} = \frac{\theta}{2\pi}$

$360\theta = 2\pi \cdot 220$

$\frac{360\theta}{360} = \frac{440\pi}{360}$

$\theta = \frac{22\pi}{18} = \frac{11\pi}{9} \approx 3.840$

Example 2: Convert 30° to radians.

$\frac{30}{360} = \frac{\theta}{2\pi}$

$\frac{360\theta}{360} = \frac{60\pi}{360}$

$\theta = \frac{\pi}{6} \approx .524$

Accelerated Algebra II: Trigonometry ~ Part II

Recall that __150__°, __210__°, and __330__° all have a reference angle of 30°. Convert these angle measures to radians.

$150° = 180° - 30°$
$150° = \pi - \frac{\pi}{6}$
$150° = \frac{5\pi}{6}$

$210° = 180° + 30°$
$210° = \pi + \frac{\pi}{6}$
$210° = \frac{7\pi}{6}$

$330° = 360° - 30°$
$330° = 2\pi - \frac{\pi}{6}$
$330° = \frac{11\pi}{6}$

Example 3: Convert 60° to radians.

$\frac{60}{360} = \frac{\theta}{2\pi}$

$\frac{360\theta}{360} = \frac{120\pi}{360}$

$\boxed{\theta = \frac{\pi}{3}}$

Recall that __120__°, __240°__°, and __300__° all have a reference angle of 60°. Convert these angle measures to radians.

$120° = 180° - 60°$
$= \pi - \frac{\pi}{3}$
$120° = \frac{2\pi}{3}$

$240° = 180° + 60°$
$= \pi + \frac{\pi}{3}$
$240° = \frac{4\pi}{3}$

$300° = 360° - 60°$
$= 2\pi - \frac{\pi}{3}$
$300° = \frac{5\pi}{3}$

Example 4: Convert 45° to radians.

$\frac{45°}{360°} = \frac{\theta}{2\pi}$

$\frac{360\theta}{360} = \frac{90\pi}{360}$

$\boxed{\theta = \frac{\pi}{4}}$

Recall that __135__°, __225__°, and __315__° all have a reference angle of 45°. Convert these angle measures to radians.

$135° = 180° - 45°$
$= \pi - \frac{\pi}{4}$
$135° = \frac{3\pi}{4}$

$225° = 180° + 45°$
$= \pi + \frac{\pi}{4}$
$225° = \frac{5\pi}{4}$

$315° = 360° - 45°$
$= 2\pi - \frac{\pi}{4}$
$315° = \frac{7\pi}{4}$

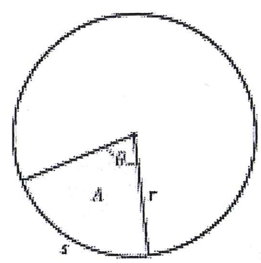
Length of an Arc and Area of a Sector

When a central angle, θ, of a circle with radius r is measured in radians, the length of the intercepted arc, s, is given by the equation

$$s = r\theta$$

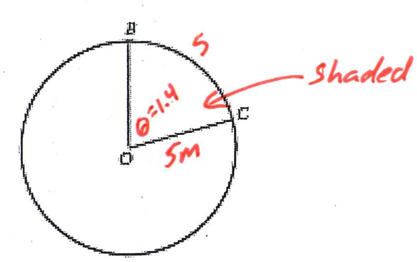
and the area of the intercepted sector, A, is given by the equation

$$A = \frac{1}{2}r^2\theta$$



Example: Circle O has a diameter of 10 m. The measure of the central angle BOC is 1.4 radians. What is the length of it intercepted arc $\overset{\frown}{BC}$?

$S = r\theta$
$S = 5(1.4)$
$S = 7.0m$



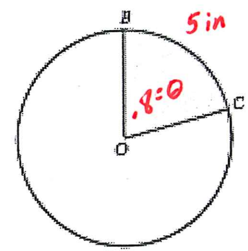
Accelerated Algebra II: Trigonometry ~ Part II

What is the area of the shaded sector?

$$A = \frac{1}{2} r^2 \theta$$
$$= \frac{1}{2}(5^2)1.4$$
$$= \frac{1}{2}(25)(1.4)$$
$$\boxed{A = 17.5\,m^2}$$

Example: If the intercepted $\overset{\frown}{BC}$ is 5 inches and the measure of the central angle BOC is 0.8 radians, what is the radius of the circle?

$$S = r\theta$$
$$\frac{5}{.8} = \frac{r(.8)}{.8}$$
$$\boxed{6.25\,m = r}$$



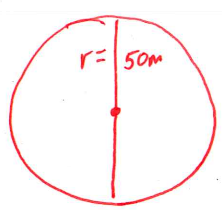
What is the area of the shaded sector?

$$A = \frac{1}{2} r^2 \theta$$
$$= \frac{1}{2}(6.25)^2(.8)$$
$$\boxed{A = 15.625\,m^2}$$

Angular Speed: The amount of rotation, or angle traveled, per unit of time.

Example: The Cosmo Clock 21 Ferris Wheel as the Cosmo World amusement park in Yokohama, Japan, has a 100 m diameter. This giant Ferris wheel, with 60 gondolas and 8 people per gondola makes one complete rotation every 15 minutes. The wheel reaches a maximum height of 112.5 m from the ground.

a. Find the speed of a person on this Ferris wheel as it is turning.



$$C = 2\pi r \quad \text{or} \quad C = \pi d$$
$$= 2\pi(50) \qquad C = 100\pi\,m$$
$$C = 100\pi\,m$$

$$\text{Speed} = \frac{C}{15\,min} = \frac{100\pi\,m}{15\,min} = 20.944\,m/minute$$

b. Find the angular speed of this person. $\frac{2\pi\,rad}{15\,min} = .419\,rad/min$